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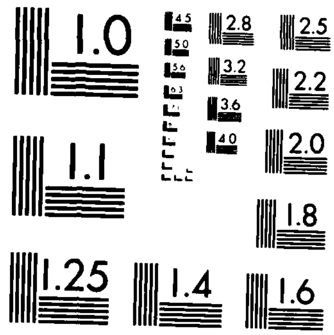

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WEAPON ACQUISITION AND ALLOCATION UNDER CONDITIONS OF TARGET UNCERTAINTY

Marc Mangel
Ronald H. Nickel

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This research contribution analyzes the problem of ordnance acquisition when the targets to be attacked are uncertain. It introduces the use of a utility function to evaluate the outcome of the attack process. Two models of the attack process are considered: The first assumes that the random target vector is attacked simultaneously with the available weapons. The second is a sequential-attack model in which targets appear one at a time and the attack process continues as long as the current target can be attacked. The expected

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It is argued that older methods of selecting weapons are biased towards special-purpose weapons and that the SIAM and SEAM models are not subject to this deficiency. ~~Furthermore,~~ In spite of being the more complicated model, the SEAM approach appears more realistic in the way that it models the attack process. An operational example is used to illustrate the problem and the SEAM approach.

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WEAPON ACQUISITION AND ALLOCATION UNDER CONDITIONS OF TARGET UNCERTAINTY

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ABSTRACT

This research contribution analyzes the problem of ordnance acquisition when the targets to be attacked are uncertain. It introduces the use of a utility function to evaluate the outcome of the attack process. Two models of the attack process are considered: The first assumes that the random target vector is attacked simultaneously with the available weapons. The second is a sequential-attack model in which targets appear one at a time and the attack process continues as long as the current target can be attacked. The expected utility for a mix of weapons for the simultaneous-attack model (SIAM) is computed as the weighted average of the expected utility of the mix of weapons against each target vector. The expected value for the sequential-attack model (SEAM) is estimated by using a simulation of the attack process.

It is argued that older methods of selecting weapons are biased towards special-purpose weapons and that the SIAM and SEAM models are not subject to this deficiency. Furthermore, in spite of being the more complicated model, the SEAM approach appears more realistic in the way that it models the attack process. An operational example is used to illustrate the problem and the SEAM approach.

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INTRODUCTION

The history of warfare shows that war is a highly uncertain business. This research contribution addresses one major uncertainty: the distribution of targets to be attacked. Procurement and allocation decisions that must be made far in advance of actual combat are based on an incomplete knowledge of what will happen when combat begins. The uncertainty is exacerbated by the long production times of most weapons, which means that unless the conflict is lengthy (for example, more than a year), one can expect to fight the next conventional war with whatever weapons are on hand when it starts. The question then is: What is the best mix of weapons to have available at the start of the conflict, given the high level of uncertainty?

This question arises at a number of different levels in the naval command structure. The highest level involves OpNav procurement decisions about what kinds of ordnance to acquire when the exact nature of future targets is unknown. The next levels pertain to allocation of ordnance. That is, once ordnance is acquired, decisions must be made regarding the kinds of weapons to stock at supply bases for the various fleets and regions. Once a given region is supplied, the problem arises again when an aircraft carrier is loaded for deployment. If a confrontation begins, the carrier will probably have to fight with the weapon mix that it loads at deployment. For this reason, the weapons mix loaded at deployment must be the most effective one possible.

This research contribution introduces new methodological concepts that can be used to help solve the problem of optimal weapon acquisition and allocation under target uncertainty. It addresses only conventional ordnance. Particular attention is paid to the mix of general-purpose (GP) and special-purpose (SP) weapons. Roughly defined, a GP weapon is one that is effective against a variety of targets under a variety of environmental conditions and defensive countermeasures. An example of a GP weapon is an "iron bomb" that follows a simple ballistic trajectory after release. An SP weapon is one that is effective against only a few targets, requires special environmental conditions and defensive countermeasures, but has a higher probability of destroying the target than the GP weapon under the same circumstances. An example of an SP weapon is a television- or laser-guided bomb. A fundamental question answered by the methodology introduced in this document involves the proportions of SP and GP weapons in a stockpile of fixed size, given that the targets to be attacked are uncertain.

Uncertainty in the distribution of targets implies an uncertainty in the scenarios in which a given mix of ordnance will be used. Thus, a mix of weapons against a particular scenario can no longer be optimized. Instead, both the variation in scenarios and how a given mix of ordnance performs against a variety of scenarios must be considered. This observation is the key to the development of the methodology

For given K_j^1, K_j^2, and ΔK_j, equation 17 provides a way that ρ_j can be determined. Table 6 shows values of ρ_j for a variety of risk premiums.

TABLE 6

VALUE OF ρ_j FOR A VARIETY OF VALUES OF THE RISK PREMIUM

k_J^1	p	k_j^2	$1 - p$	ΔK_j^*	ρ_j
0	1/2	100	1/2	5	.004
				10	.008
				15	.0127
				20	.018
				25	.024
0	1/2	1,000	1/2	50	.0004
				100	.0008
				150	.00127
				200	.0018
				250	.0024
				300	.0033
				350	.0046

Note: ΔK_j is the amount by which the certain outcome is reduced to render the decision maker indifferent to the certain and uncertain outcomes.

The utility function (equation 15) has a number of useful properties. First, observe that

$$\lim_{\rho_j \to 0} \frac{1}{\rho_j} u(K_j) = K_j \tag{18}$$

so that the expected value of $(1/\rho_j)\, u(k_j)$ is, for small enough ρ_j, an implicit measure of the expected number of targets destroyed.

To capture the uncertainty in the outcomes of attacks, some measure of preference in outcomes must be determined. Here, the concept of a "utility function" [2] is extended to military combat situations. The utility function $u(x)$ is a single-valued function that can be used to specify preferences in outcomes. In particular, for military problems $u(x)$ is assumed to have the property of risk aversion. That is, if K_j^1 and K_j^2 are two choices for K_j, occuring with probability p and $1 - p$, then

$$pu\left(K_j^1\right) + (1 - p)u\left(K_j^2\right) < u\left[pK_j^1 + (1 - p)K_j^2\right] . \tag{14}$$

If the equality holds, the decision maker is called risk-neutral; otherwise, the decision maker is risk-averse. The utility function chosen in this document is

$$u(K_j) = 1 - \exp(-\rho_j K_j) \tag{15}$$

where ρ_j is a parameter. This utility function is chosen mainly for mathematical convenience. Although it has not been thoroughly investigated and the precise details of the mix might vary, it is reasonable to assume that other concave utility functions [such as $u(x) = \log(1 + x)$] would give results similar to the ones presented here.

Clearly, equation 14 is satisfied by equation 15, with a strict inequality. The parameter ρ_j is determined as follows. Consider the amount ΔK_j that the certain outcome must be reduced so that there is no preference between certain and uncertain outcomes. That is, define ΔK_j by:

$$pu\left(K_j^1\right) + (1 - p)u\left(K_j^2\right) = u\left(pK_j^1 + (1 - p)K_j^2 - \Delta K_j\right) . \tag{16}$$

The value of ΔK_j is called the risk premium. For the utility function (equation 15), equation 16 becomes

$$p \exp\left\{-\rho_j K_j^1\right\} + (1 - p) \exp\left\{-\rho_j K_j^2\right\} = \exp\left\{-\rho_j\left[pK_j^1 + (1 - p)K_j^2 - \Delta K_j\right]\right\} . \tag{17}$$

$$\sum_{i=1}^{W} N_i = N \quad .$$

WAP2 indicates that the determination of the optimal mix proceeds in the following fashion:

1. Specify the constraint N.

2. Choose a mix $\{N_i\}$ subject to $\sum N_i = N$.

3. Determine the optimal allocation from this mix against a target vector t.

4. Average the values obtained in step 3 over all possible target vectors.

5. Optimize the value obtained in step 4 over all possible mixes $\{N_i\}$.

Because $V(\{N_i\} \mid \{t\})$ involves an optimization, WAP2 poses a two-stage optimization problem and is the appropriate method for dealing with target uncertainty. The actual method for calculating $V(\{N_i\} \mid \{t\})$ must be given in order for the methodology inherent in WAP2 to be applied. The next three sections describe the computation of $V(\{N_i\} \mid \{t\})$.

UTILITY AND VALUE

One natural choice for $V(\{N_i\} \mid \{t\})$ is the expected number of targets destroyed when the mix of weapons is $\{N_i\}$; however, a more general methodology can be used to capture the effects of the uncertainty in the outcome of an attack. That is, observe that even when the target vector is known precisely, the results of an attack using a mix $\{N_i\}$ are random variables. In fact, if T_j targets of type j are attacked with n_{ij} weapons of type i, and if attacks are assumed to be independent random events, the number of targets destroyed follows a binomial distribution:

$$\Pr\{K_j = k\} = \binom{T_j}{k} \left[p_{kj}(n_{ij}, T_j)\right]^k \left[1 - p_{kj}(n_{ij}, T_j)\right]^{T_j - k} \tag{13}$$

where $p_{kj}\,(n_{ij}, T_j)$ is given by equation 10.

TWO-STAGE OPTIMIZATION

This section describes, in general, the correct approach for solving problems with target uncertainty. Details are worked out in the following sections.

Imagine a number (possibly infinite) of target vectors $\{t^1\}$, $\{t^2\}$,... with some distribution function that specifies the probability of having to attack target vector $\{t^i\} = \left(t_1^i, t_2^i, \ldots, t_Q^i\right)$. These target vectors correspond to different scenarios. Next, imagine that a mix of weapons $\{N_i\}$ is specified.* This mix has a certain "value" against each of the target vectors. (The precise meaning of "value" is described in detail in the next three sections.) Let $V\left(\{N_i\} \mid \{t^k\}\right)$ denote the value of weapon mix $\{N_i\}$ against target proportion vector $\{t^k\}$. In general, $V\left(\{N_i\} \mid \{t^k\}\right)$ will be determined by some kind of optimization procedure (something similar to WAP1). That is, for a given $\{t\}$ vector and a given mix $\{N_i\}$, weapons are allocated optimally to targets. The ultimate value of the mix $\{N_i\}$ is then found by averaging $V\left(\{N_i\} \mid \{t\}\right)$ over the distribution on $\{t\}$:

$$V(\{N_i\}) = E_{\{t\}}[V(\{N_i\} \mid \{t\})] \quad . \tag{12}$$

In equation 12, $V(\{N_i\})$ is the value of weapons mix $\{N_i\}$, and $E_{\{t\}}$ denotes an expectation over all possible target vectors. Finally, imagine that the total number of weapons is constrained to be N. The optimal mix of weapons is then found by solving the problem

Weapon Allocation Problem Two (WAP2):

$$\max_{\{N_i\}} E_{\{t\}} [V(\{N_i\} \mid \{t\})]$$

subject to $N_i \geq 0$ for all i and

---

* Although the fundamental variables are components, it is easier to describe the methodologies in terms of weapons. Thus, the term "weapons" rather than "components" will be used in the next few sections.

Equations similar to equation 11 are the basic model (SABRE MIX) that the Air Force uses to plan weapon acquisition and allocation [1]. Indeed, it appears reasonable to try to determine the optimal mix of weapons by using the following optimization scheme. Suppose that the total number of weapons to be acquired is N. Then consider the problem

Weapons Allocation Problem One (WAP1):

$$\max_{\{n_{ij}\}} \sum_{j=1}^{Q} T_j \left[1 - \exp\left(-\sum_{i=1}^{W} n_{ij} P_{ij} / T_j \right) \right]$$

Subject to $n_{ij} \geq 0$ for all i,j and

$$\sum_{j=1}^{Q} \sum_{i=1}^{W} n_{ij} = N \quad .$$

The solution to WAP1 provides a set $\{n_{ij}^*\}$ where n_{ij}^* is the number of weapons of type i *to be used against targets of type* j. This set, however, is in no sense optimal for the true operational problem. In the appendix it is proved that if an SP weapon exists for each target type, the solution to WAP1 consists only of SP weapons; no GP weapons will ever be selected. Second, the entire formulation of WAP1 is somewhat inappropriate. That is, one obtains as the solution a set $\{n_{ij}^*\}$ where n_{ij}^* is the number of type-i weapons to be used against only type-j targets. The operational picture here is that one sets aside a "bundle" of n_{ij}/T_j weapons of type i for each of the expected targets of type j, and these weapons are not used until a target of type j appears. It is as if all weapons are used simultaneously, which does not happen in a combat situation. Third, the formulation WAP1 cannot be used to take target uncertainty into account. Even averaging the resulting optimal weapon vectors over all target vectors is incorrect, because it does not make sense in terms of the ultimate weapons mix. (What is an "average optimal" mix?) Because the WAP1 formulation picks only SP weapons, the mix determined by averaging has only SP weapons in it. The choice of GP weapons as a kind of insurance against uncertainty will not occur. A different formulation is needed.

Assuming that the deliveries constitute independent random events, the probability of destroying the target with n weapons is

$$1 - (1 - \hat{P}_{ij})^n = 1 - \exp [n \log (1 - \hat{P}_{ij})] \quad . \tag{6}$$

In light of the form of equation 6, it is helpful to define

$$P_{ij} = -\log (1 - \hat{P}_{ij}) \tag{7}$$

so that the probability of destroying the target with n weapons is now

$$1 - \exp (-nP_{ij}) \quad . \tag{8}$$

Now imagine that n_{ij} weapons of type i are used against T_j targets of type j. To maximize the expected number of targets destroyed, the weapons should be allocated uniformly to the targets. Thus, if K_j denotes the number of type-j targets killed by the n_{ij} weapons, the expected value of K_j is

$$E\{K_j\} = T_j\left[1 - \exp \left(-\sum_{i=1}^{W} n_{ij}P_{ij}/T_j\right)\right] \tag{9}$$

where $E\{x\}$ denotes mathematical expectation. Because the quantity in brackets on the right side of equation 9 will appear throughout the document, it is useful to define it by

$$P_{kj}(n_{ij}, T_j) = 1 - \exp\left(-\sum_{i=1}^{W} n_{ij}P_{ij}/T_{ij}\right) \quad . \tag{10}$$

This quantity is simply the probability of destroying one type-j target when n_{ij}/T_j weapons of type i are used on it.

The total expected number of targets destroyed is thus

$$E\{K\} = \sum_{j=1}^{Q} T_j\left[1 - \exp \left(-\sum_{i=1}^{W} n_{ij}P_{ij}/T_j\right)\right] \quad . \tag{11}$$

COMPONENTS AND WEAPONS

In actual practice, "weapons" are not acquired or allocated. Instead, ordnance or components of weapons are acquired or allocated. Components consist of the actual physical delivery device (usually an aircraft or missile) and the components (e.g., bomb bodies, fins, or seeker heads).

In general, more than one set of weapons can be created from a given set of components. This is an added complication in the problem. In reality, an optimal mix of components should be sought rather than an optimal mix of weapons. The difference between weapons and components will become clear as the methodology of this document is developed. For simplicity, dependence upon delivery device will be suppressed.

The notation needed for components and weapons is the following. The vector of components is denoted by $\{C_i\}$ where C_i is the number of the i^{th} kind of component. The vector of weapons is denoted by $\{N_i\}$ where N_i is the number of the i^{th} kind of weapon. The total numbers of components and weapons are denoted by C and N, respectively. In the notation $\{N_i\}$, i runs from 1 to W, where W is the total number of types of weapons.

SINGLE- AND MULTIPLE-SHOT KILL PROBABILITIES

The piece of data fundamental to all of this work is the single-shot kill probability $\hat{P}_{ij}$ defined by

$$\hat{P}_{ij} = \text{probability a single weapon of type } i \text{ destroys a target of type } j \ . \quad (4)$$

When n weapons are delivered against the target, the probability of destroying the target is

$$1 - \text{probability \{none of the } n \text{ weapons destroys the target\}} \ . \quad (5)$$

TABLE 5

SUMMARY OF NOTATION

Symbol	Meaning
$\{T_j\}$	Vector of target types; T_j is the number of targets of type j; target vectors will be indexed by superscripts, i.e., $\{T^k\}$ is the k^{th} target vector
$\{t_j\}$	Vector of target proportions; t_j is the fraction of targets of type j
T	Total number of targets
$\{C_i\}$	Vector of component types; C_i is the number of components of type i
C	Total number of components
$\{N_i\}$	Vector of weapons types; N_i is the number of weapons of type i
N	Total number of weapons
$\hat{P}ij$	Single-shot kill probability of a weapon of type i against a target of type j
P_{ij}	Transformation of the $\hat{P}_{ij}$ given by $P_{ij} = -\log (1 - \hat{P}_{ij})$
K_j	Number of j-type targets destroyed
$P_{kj} (n_{ij}, T_j)$	Probability of destroying one type-j target when n_{ij} weapons of type i are used against T_j type-j targets
$V(\{N_i\} \mid \{t^k\})$	Value of weapons mix $\{N_i\}$ against target proportion vector $\{t^k\}$
$V(\{N_i\})$	Value of weapons mix $\{N_i\}$ averaged against all target vectors
U	Utility associated with one simulation run
$\langle \cdot \rangle$	Average over many simulation iterations
U_s	Super utility function

MODELING ISSUES AND APPROACHES

This section discusses a number of issues that are germane to the ultimate problem of choosing a mix of weapons when there is uncertainty about the targets.

THE NATURE OF THE TARGET

For the purposes of this document, the target has three features: (1) the actual, physical target such as a bridge, road, or tank, (2) the environmental conditions when the target is about to be attacked, and (3) the defensive countermeasures associated with the target. Even if the targets that are to be attacked are known precisely, the associated weather and defensive countermeasures will always be uncertain at the time of weapon acquisition. For this reason, there will always be uncertainty in the distribution of targets.

With this picture, consider the vector $\{T_j\}$ of targets where

$$\{T_j\} = (T_1, T_2, \ldots, T_Q) \quad , \tag{1}$$

T_j is the number of targets of type j, and there are Q different kinds of targets. It is also convenient to introduce the total number of targets T given by

$$T = \sum_{j=1}^{Q} T_j$$

and the vector $\{t\}$ characterizing the proportion of targets

$$\{t_j\} = (T_1/T, T_2/T, \ldots, T_Q/T) \tag{2}$$

$$= (t_1, t_2, \ldots, t_Q) \tag{3}$$

so that t_j is the proportion of targets of type j. (Table 5 contains a summary of all notation used in this document.)

TABLE 4

PROBABILITIES OF KILL

Target type	Visibility condition	Weapon	Number of passes	Probability of destroying the target[a]
Tank	High	LGB-83	1	.80
Tank	High	Walleye	1	.75
Tank	Medium	4 Mk 82s	1	.50
Tank	Low	4 Snakeyes	1	.35
Tank	Radar	2 Mk 83s	1	.20
Bridge	High	Walleye	1	.75
Bridge	High	LGB-83	1	.50
Bridge	Medium	2 Mk 83s	1	.10
Bridge	Radar	1 Mk 84	2	.05
Building	High	Walleye	2	.80
Building	Medium	4 Mk 82s	1	.30
Building	Medium	2 Mk 83s	1	.25
Building	Low	4 Snakeyes	1	.20
Building	Radar	4 Mk 82s	1	.10
Runway	High	LGB-83	3	.60
Runway	Medium	4 Mk 82s	6	.50
Runway	Low	4 Snakeyes	6	.45
Runway	Radar	4 Mk 82s	6	.20
POL farm	High	LGB-83	10	.70
POL farm	Low	4 Snakeyes	3	.60
POL farm	Radar	4 Mk 82s	4	.45

[a]These values are for illustrative purposes only, and do not reflect true capabilities.

TABLE 3

POSSIBLE WEAPON LOADS

Load	Components used
4 Mk 82s	4 Mk 82s
4 Snakeye Mk 82s	4 Mk 82s, 4 Snakeye fins
1 LGB-83	1 Mk 83, 1 LGB kit
2 Mk 83s	2 Mk 83s
1 Mk 84	1 Mk 84
1 Walleye	1 Walleye

If there were no uncertainty about environmental conditions and target types, the tendency would be to choose a mix dominated by LGBs and Walleyes. This is clearly not a good idea when environmental and target uncertainty is considered. The following sections develop a methodology for choosing a mix that takes environmental and target uncertainty into account.

TABLE 2

PROBABILITY OF VARIOUS VISIBILITY CONDITIONS FOR EACH SCENARIO

	Probability of			
Scenario	High visibility	Medium visibility	Low visibility	Radar only
1	.40	.20	.30	.10
2	.70	.20	.05	.05
3	.50	.30	.15	.05
4	.25	.45	.25	.05

The ordnance components in this example are Mk 82, 83, and 84 bomb bodies, fins for the bomb bodies, Snakeye fins, laser-guided bomb (LGB) kits, and Walleye. These basic components can be made into different weapons, called weapon loads (table 3). The components are subject to a volume constraint; here it is taken to be

$$\text{Number Mk 82s} + 2 \times \text{number Mk 83s} + 4 \times \text{number Mk 84s} + .5 \times \text{number Snakeye fins} + 1.5 \times \text{number of LGB kits} + 10 \times \text{number of Walleyes} \leq 1{,}000 \quad .$$

This constraint shows the kinds of trade-offs that one must think about. For example, if one Walleye is added to the mix, the number of Mk 82s is reduced by 10. (Other choices, of course, are possible. For example, one Walleye has the same volume as 2 Mk 84s and 2 Mk 82s.) In addition to the total volume constraint, three other constraints are applied, all of which were chosen arbitrarily for illustrative purposes:

$$\text{Number of Mk 82s} \leq 300$$

$$\text{Number of Mk 84s} \geq 40$$

$$\text{Number of Walleyes} \geq 20.$$

One further consideration in decisions regarding the weapon mix is the probability that a given weapon destroys a given target. These data are shown in table 4, which ranks the kill probabilities of different weapons against the various target types.

AN OPERATIONAL EXAMPLE

The detailed operational example described below illustrates the origin of the target uncertainty problem, the kinds of data needed to solve the problem, and how the solution is obtained. The generic values of the single-shot probability of kill for each weapon type are for illustration only.

The example contains five kinds of targets: bridges, tanks, buildings, runways, and petroleum, oil, and lubricant (POL) farms. The uncertainty concerning these targets involves the proportion of each kind within the mix. Table 1 shows the proportions of different target types in each of four possible scenarios. That is, it is presumed that only one of the four scenarios will occur, but which one is not known at the time of ordnance acquisition. A second interpretation is that the proportions in table 1 represent long-run frequencies of target types. In three of the four scenarios, tanks are the main target type. Choosing an ordnance mix that is highly effective against tanks but not against bridges or buildings, for example, can lead to disastrous results if scenario four represents the true situation.

TABLE 1

PROPORTIONS OF TARGET TYPES IN THE FOUR SCENARIOS

		Proportion of target type				
Scenario	Probability of that scenario	Bridge	Tank	Building	Runway	POL farm
1	.4	.22	.43	.09	.09	.17
2	.2	.05	.54	.03	.16	.22
3	.2	.02	.82	.06	.04	.06
4	.2	.23	.15	.24	.15	.23

Environmental conditions and defensive countermeasures are associated with the vector of targets. Four environmental conditions are considered: high, medium, and low visibility and radar-only attack. For simplicity, no defensive countermeasures are considered in this example. Table 2 shows the probability of each visibility condition for each scenario.

presented here. Although it may appear trivial and obvious, it has not received the attention it deserves. Most existing procedures for weapon acquisition are highly suboptimal, because they work with a scenario obtained by averaging over the unknown target vectors. The trouble is that the average scenario may not be similar to any of the scenarios from which it arises.

The next section describes a detailed operational example. The third section discusses various modeling issues and approaches. The fourth and fifth sections describe the expected utility models and apply them to the operational example. The final section contains conclusions and a discussion of other aspects of the problem.

Second, suppose that $K_j^1 = k$ and $K_j^2 = 0$. Then the function

$$\begin{aligned} \Delta u &= u(pk) - [pu(k) + (1 - p)u(0)] \\ &= (1 - p) - \exp(-\rho_j pk) + p \exp(-\rho_j k) \end{aligned} \tag{19}$$

is a monotonically increasing function of ρ_j, rising to

$1 - p$ as $\rho_j \to \infty$.

Third, ρ_j is nearly linear in K_j and ΔK_j so that it is easy to estimate a value of ρ_j for values of K_j and ΔK_j not shown in table 6.

Fourth, consider the expected value of $u(K_j)$. Let p_k be the probability of kill. Then

$$\begin{aligned} E\{u(K_j)\} &= \sum_{\ell=0}^{T_j} (1 - \exp(-\rho_j \ell)) \binom{T_j}{\ell} p_k^{\ell} (1 - p_k)^{T_j - \ell} \\ &= 1 - \left[1 - p_k(1 - \exp(-\rho_j))\right]^{T_j} . \end{aligned} \tag{20}$$

The last equation follows by the use of the moment generating function for a binomial random variable. Observe that as $\rho_j \to \infty$,

$E\{u(K_j)\} \to 1 - (1 - p_k)^{T_j}$, which is the probability that at least one target is destroyed.

It is now possible to specify $V(\{N_i\} \mid \{T_j\})$ completely. In particular, if $\{T_j\} = (T_1, T_2, \ldots, T_Q) = T(t_1, t_2, \ldots, t_Q)$, then one choice for $V(\{N_i\} \mid \{T\})$ is

$$V(\{N_i\} \mid \{T\}) = \max_{\{n_{ij}\}} \sum_{j=1}^{Q} \left\{1 - [1 - p_{kj}(1 - \exp(-\rho_j))]^{T_j}\right\} \tag{21}$$

$$\text{subject to} \quad \sum_{j=1}^{Q} n_{ij} = N_i, \quad n_{ij} \geq 0 \text{ for all } i,j$$

$$\text{where} \quad p_{kj} = 1 - \exp\left(- \frac{1}{T_j} \sum_{i=1}^{W} n_{ij} P_{ij}\right).$$

Equation 21 specifies the value function for a particular operational model. This model is discussed in detail in the next section.

EXPECTED UTILITY MODELS

This section describes two specific methods for the calculation of the value function $V(\{N_i\})$, and thus the optimal mix $\{N_i^*\}$.

SIMULTANEOUS-ATTACK MODEL

The first method uses the expected utility given in equation 21. That is, the value V of the optimal mix of weapons is determined by solving the optimization problem in the simultaneous-attack model (SIAM):

$$V = \max_{\{N_i\}} E_{\{T\}} [V(\{N_i\} \mid \{T\})]$$

$$\text{subject to } \sum_{i=1}^{W} N_i = N \quad N_i \geq 0 \quad .$$

The two-stage optimization problem in SIAM is the correct formulation of the problem. An interchange of maximization and expectation will give an incorrect formulation of the problem. This will be true even if one works simply with expected kills rather than a value function. That is, suppose that the total size of the mix is N. The correct formulation of the problem using expected kills is then

$$V = \max_{\substack{\{N_i\} \\ \sum N_i = N}} E_{\{T\}} \left\{ \begin{array}{l} \max\limits_{\{n_{ij}\}} EK\,(n_{ij} \mid \{T\}) \\ \text{subject to } \sum\limits_j n_{ij} = N_i \text{ for all } i \end{array} \right\} .$$

Here $EK\,(n_{ij} \mid \{T\})$ is the expected number of targets killed when $\{T\}$ is the target vector and n_{ij} weapons of type i are used against targets of type j. The incorrect formulation exchanges the roles of maximization and expectation and uses

$$V_{inc} = \max_{\{n_{ij}\}} E_{\{T\}}[EK\,(n_{ij} \mid \{T\})]$$

$$\sum_i \sum_j n_{ij} = N \quad .$$

The two-stage nature of the optimization problem is seen clearly in the SIAM formulation. A number of other properties of the SIAM model are discussed below.

First, the SIAM model is similar to the expected kill model (equation 11) in the assumption that all weapons are used simultaneously. If the solution to the SIAM model is $\{n^*_{ij}\}$, it has the following interpretation. For each target vector $\{T\}$, n^*_{ij} weapons of type i are set aside to be used against targets of type j. In this sense, the SIAM model is highly unrealistic because certain weapons are not put aside for use only when a type-j target appears. In particular, such a model will overemphasize the value of SP weapons. For example, in a situation in which targets appear one at a time and the attack process continues as long as a target can be attacked, a mix of weapons that is heavily weighted towards SP may have a higher expected utility in the SIAM model than a mix with fewer SP weapons. On the other hand, in the sequential-attack process, such a mix may fail much sooner than the mix with fewer SP weapons. A methodology that avoids the problem of simultaneous use of weapons is introduced below.

An advantage of the SIAM model is that it is a relatively straightforward nonlinear programming problem. In particular, the "inner problem," the determination of $E_{\{T\}}[V(\{N_i\} \mid \{T\})]$, can be done using a nonlinear programming algorithm [3]. The solution of the "outer problem," the maximization over $\{N_i\}$, can be found using a finite differencing procedure.

Numerical experiments indicate that the following scheme provides an excellent approximation to $V(\{N_i\} \mid \{T\})$ [4]. For a given vector $\{T\}$ of targets, divide GP weapons uniformly among all targets, and divide SP weapons uniformly among the targets they were designed to attack. By using this approximation scheme, the two-stage optimization problem can be reduced to a one-stage problem (the outer optimization problem). Examples using SIAM are given in [4].

A number of difficulties are associated with the SIAM model. The basic, and most severe, difficulty is that any SIAM model is highly unrealistic because of the assumption that all weapons are used simultaneously. A second difficulty is that one must specify the total number of targets to be attacked, as well as the target vectors. A new methodology that avoids the simultaneous attack assumption is needed.

SEQUENTIAL-ATTACK MODEL

An alternative to the simultaneous-attack model associated with the expected utility methodology is one in which targets are attacked sequentially. The methodology for the sequential-attack model (SEAM) involves a simulation technique that is described in this section. Because of the flexibility offered by simulation, this discussion considers the mix of components rather than weapons.

The fundamental features of the simulation for computing the utility of a particular mix of components $\{C_i\}$, are as follows. First, the pool of targets is infinite, with the proportions of different kinds of targets uncertain. That is, the vector $\{t\} = (t_1, t_2, \ldots, t_Q)$ is uncertain. It is assumed that the probability distribution of $\{t\}$ is known.

Second, the value function $V(\{C_i\} \mid \{t\})$ is computed according to the following prescription. Targets appear from the pool one at a time, chosen according to the probability distribution of $\{t\}$. When a target appears, the weapon with the highest single-shot probability of kill that can be constructed from the current mix of components and that can be used in the randomly determined environment is then used against the target. The vector of components is decreased according to the components that were used to attack the target. When a target appears and no effective weapon can be built from the current mix of components, the simulation iteration ends. The utility associated with the simulation iteration is

$$U = \sum_{j=1}^{Q} (1 - \exp(-\rho_j K_j)) \tag{22}$$

where K_j is the number of j-type targets destroyed during the simulation iteration. Because U is a random variable, a large number of simulation iterations were performed for each component vector $\{C_i\}$. The number of iterations was chosen so that the coefficient of variation of U was 1.0 percent. The value function is then

$$V(\{C_i\} \mid \{t\}) = \langle U \rangle \tag{23}$$

where $\langle x \rangle$ denotes an average over the simulation iterations. In addition to $\langle U \rangle$, the value of $\sigma^2 = \langle U^2 \rangle - \langle U \rangle^2$ was also recorded for each mix of components.

The third feature of the simulation is that it has many possible modifications. For example, targets could appear in groups of random or known size rather than one at a time. Targets could be attacked until destroyed rather than once and then left alone. Instead of stopping the simulation when a target appears and cannot be attacked, a "disutility" could be accumulated. Uncertain kill probabilities can easily be included, as can targets of differing values (by using $u_j(K) = A_j\ (1 - \exp(-\rho_j K))$ as the utility of K targets of type j destroyed).

A successive linear programming algorithm [5] has been developed that iterates on the component vectors $\{C_i\}$ until no further improvement in the utility is possible. The methods of stochastic approximation may also be applicable to solving the SEAM problem [6].

CHOICES OF MEASURES OF EFFECTIVENESS

The value functions $V(\{N_i\} \mid \{t\})$ or $V(\{C_i\} \mid \{t\})$ described in the previous section contain an explicit measure of effectiveness (MOE)—the utility associated with the destruction of K_j targets of type j, $1 - \exp(-\rho_j K_j)$. For a given scenario (that is, a given vector $\{t\}$), this utility function provides a general MOE and captures the effects of uncertainty.

There is one uncertainty for which another MOE could be considered. This is the uncertainty in scenarios (i.e., the uncertainty in the $\{t\}$ vectors). Both methodologies introduced for the calculation of $V(\{N_i\})$ assume a simple, linear MOE since the value of a set of weapons is computed by

$$V(\{N_i\}) = E_{\{t\}}[V(\{N_i\} \mid \{t\})] \quad . \tag{24}$$

One other possible MOE is to replace equation 24 with a utility-of-the-utility model in which

$$V(\{N_i\}) = E_{\{t\}}\{U_s[V(\{N_i\} \mid \{t\})]\} \tag{25}$$

where $U_s(x)$ is a "super" utility function. A natural choice would be $U_s\ (V) = 1 - \exp(-\hat{\rho}V)$ where $\hat{\rho}$ is a parameter. In principle, $\hat{\rho}$

can be determined in a manner analogous to the manner in which the ρ_j are determined (although the phraseology of the questions is not as clear).

Another choice would be

$$V(\{N_i\}) = E_{\{t\}}[V(\{N_i\} \mid \{t\})] - \alpha\, E_{\{t\}}\left\{V(\{N_i\} \mid \{t\}) - E_{\{t\}}[V(\{N_i\} \mid \{t\})]\right\}^2 \tag{26}$$

The second term on the right side of equation 29 is the variance of $V(\{N_i\} \mid \{t\})$ across target vectors multiplied by a scaling factor α. As α increases, $V(\{N_i\})$ decreases if there is great variation in $V(\{N_i\} \mid \{t\})$ for different $\{t\}$ vectors. Some simple examples using equation 26 are found in [4].

APPLICATION OF THE METHODOLOGY TO THE OPERATIONAL EXAMPLE

The simulation methodology will now be applied to the operational example described in the second section. Recall that the components consist of LGB kits, Mk 82, 83, and 84 bomb bodies, Snakeye fins, and Walleyes. The initial mix of components was

Component	Number
LGB kit	40
Mk 82	200
Mk 83	60
Mk 84	50
Snakeye fin	20
Walleye	40

The simulation described previously was used to evaluate the utility of the mix of components in the four scenarios. The results are shown below:

Scenario	Average utility
1	4.62
2	4.20
3	4.09
4	4.54

The average of the utility, taken over scenarios, is 4.41 and the standard deviation in the utility (averaged over simulations) is 0.022. Table 7 gives the average kills by scenario, and table 8 gives the average loads used by scenario.

TABLE 7

AVERAGE KILLS BY SCENARIO FOR THE INITIAL MIX

Target type	Scenario 1	Scenario 2	Scenario 3	Scenario 4
Bridge	3.57	1.15	0.72	1.79
Building	1.86	0.86	2.17	2.71
POL farm	4.43	4.23	2.23	3.54
Runway	1.65	3.11	1.27	2.01
Tank	10.73	14.6[illegible]	36.24	2.43

TABLE 8

AVERAGE LOADS USED BY SCENARIO FOR THE INITIAL MIX

Load	Scenario 1	Scenario 2	Scenario 3	Scenario 4
4 Mk 82s	13.83	12.87	24.19	14.42
4 Snakeye Mk 82s	3.44	1.98	4.05	2.86
1 LGB-83	11.31	14.91	25.53	4.00
2 Mk 83s	7.74	1.88	8.94	4.22
1 Mk 84	4.33	0.24	0.40	2.24
1 Walleye	6.90	7.37	10.41	3.43

The successive linear programming algorithm mentioned above was used to determine the optimal mix in six iterations, as shown here.

Component	Number
LGB kit	90
Mk 82	270
Mk 83	100
Mk 84	40
Snakeye fin	70
Walleye	20

The utility of this optimal mix in the four scenarios is

Scenario	Average utility	Percent improvement over initial mix
1	4.94	7
2	4.50	7
3	4.43	8
4	4.84	7

The average of the utility, taken over scenarios, is 4.73, and the standard deviation is 0.024. Table 9 shows the average number of kills achieved by the optimal mix, and table 10 shows the average number of each load used by scenario. Note that this mix is uniformly better, that is, it does better than the initial mix in each of the four scenarios both for utility values and for targets killed.

These results are best understood through a review of table 4, where the probability-of-kill data are given. The Mk 82s and 83s are the most general-purpose ordnance, whereas the Walleye and Mk 84 are the most specialized weapons in this example. Also in this example, one Walleye has five times the volume of a Mk 83 and ten times the volume of a Mk 82. The Mk 84 volume is two times the volume of a Mk 83 and four times the volume of a Mk 82.

TABLE 9

AVERAGE KILLS BY SCENARIO FOR THE OPTIMAL MIX

Target type	Scenario 1	2	3	4
Bridge	5.20	1.76	0.98	2.44
Building	2.91	1.24	3.20	3.78
POL farm	7.31	7.91	3.69	4.87
Runway	2.94	5.39	2.23	3.36
Tank	15.60	24.03	52.80	3.67

TABLE 10

AVERAGE LOADS USED BY SCENARIO FOR THE OPTIMAL MIX

Load	Scenario 1	2	3	4
4 Mk 82s	17.29	15.62	31.82	16.38
4 Snakeye Mk 82s	8.74	6.01	11.87	7.24
1 LGB-83	18.69	29.22	47.08	6.31
2 Mk 83s	10.06	2.76	8.90	5.20
1 Mk 84	6.29	0.42	0.46	2.71
1 Walleye	9.00	9.76	5.46	4.53

The number of LGB kits in the optimal mix is more than twice the number in the initial mix. (This also entails an increase of 40 Mk 83s for use with the kits). For this example, the LGB is a relatively general-purpose (at least for high visibility), highly effective weapon, and an increase in LGBs in the optimal mix is reasonable.

The Mk 82 is the only true general-purpose ordnance in this example in that it can be used against all targets. The optimal mix contains about 35 percent more Mk 82s than the initial mix; this is a kind of insurance that one has the capability to attack any target that may appear.

The increase in the number of Mk 83s from the initial to optimal mix nearly matches the increase in LGB kits. It is presumed then that these additional Mk 83s will be used as LGBs. This statement must be caveated because of the environmental conditions. For example, if bridges are attacked with medium visibility, the Mk 83s will be used alone rather than as LGBs.

Relative to Mk 82s and 83s, the Mk 84 is a high-volume special-purpose ordnance used only for radar attacks on bridges. For these reasons, the optimal mix contains ten fewer Mk 84s than the initial mix. The constraint on the minimum number of Mk 84s is binding in this case.

Because the Snakeye can be used against four of the five target types, it is also a relatively general-purpose weapon. It, too, can provide a kind of insurance that some capability exists for attacking the next target that appears.

The Walleye is a high-volume, relatively special-purpose weapon. The number of Walleyes in the optimal mix is half of the number in the initial mix; the minimum constraint is also binding in this case. Observe that two LGBs give the same probability of destroying a bridge as one Walleye, and 20 Mk 82s give a slightly higher probability of destroying a building than the two Walleyes. When volume considerations are included, it is best to choose the weapons with more general capabilities.

CONCLUSIONS AND DISCUSSION

This research contribution introduces a new methodological concept for dealing with uncertainty in allocation and acquisition problems. The key idea here is that the value of a specified mix of weapons must be computed by averaging over possible target scenarios. The values associated with different mixes are then compared. If the variation over scenarios is not taken into account, the acquisition process will lead to mixes of weapon components that perform poorly against the average over scenarios.* A second, but not as important, methodological concept introduced here is the use of a utility function of the number of targets destroyed, rather than simply an expected kill model. The use of a utility function has two advantages. First, it reflects the risk aversion inherent in military operations. Second, it provides a natural way of incorporating uncertainty into calculations.

Two methods for computing the value $V(\{C_i\})$ of a set of components are described here. The first is an expected utility model that corresponds to a simultaneous attack, i.e., all weapons are used. The optimal mix of weapons is determined by a two-stage optimization procedure in which a certain set of weapons is associated with a certain target type. The second method for computing the value of a mix of components is based on a sequential-attack model in which targets appear randomly according to some distribution and continue to be attacked until either all components are used up or a target appears and it can not be attacked (i.e., there are no components that can be used to make a weapon to attack this target). The sequential-attack model was studied by using a simulation procedure. The simultaneous-attack model overestimates the value of special-purpose weapons by assuming that all weapons can be used.

A number of modifications and additional aspects of the work are discussed below.

UNCERTAIN SINGLE-SHOT PROBABILITY OF KILL OR TARGET TYPE

This work assumes that the single-shot probability of kill $\hat{P}_{ij}$ was known with certainty. This may not be true for at least two reasons. First, for many of the newer weapons, the value of P_{ij} in combat can be inferred only from the existing test data in noncombat

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* This idea is embodied in the general observation that if x is a random variable and E_x denotes the expectation over x, then

$$E_x\left\{\max_u h(x,u)\right\} \geq \max_u E_x\{h(x,u)\}.$$ Although this is "obvious," its importance for ordnance acquisition has gone virtually unnoticed.

situations. For this reason, there is considerable uncertainty in the actual value of $\hat{P}_{ij}$. Second, the value of the single-shot kill probability also depends upon the countermeasures (CMs) taken to defend the target. Some CMs are purely passive and affect all of the $\hat{P}_{ij}$--for example, hardening a site. Other CMs affect some weapons and not others. For example, a SAM site will not affect the ballistic flight of an iron bomb but will affect the flight of a smart weapon that must be guided by someone inside the attacking aircraft.

These considerations are easily taken into account. In particular, simply add a distribution on $\hat{P}_{ij}$ to the problem. This addition presents no conceptual difficulty and little computational difficulty.

Similarly, to take into account different defensive countermeasures, simply extend the definition of the target vector. That is, think of a "target" as consisting of a target type with a certain countermeasure in a certain environmental condition. Once again, there is no conceptual or computational difficulty with doing this.

SUBTLETIES IN CHOOSING WEAPONS

There has been an implicit assumption in this work that when a target appears, the weapon with the highest kill probability will be used against it. Although this approach is probably reasonable, it is by no means obviously the best one. To see this, consider a problem with three targets, four weapons, and the following kill-probability matrix.

	Target		
Weapon	1	2	3
1	.5	.5	.5
2	.9	.7	.0
3	.0	.9	.0
4	.0	.0	.9

Suppose that a target of type 1 appears. The first impulse would be to use weapon 2, the SP weapon for target 1. But, if many target 2s are expected in the future and the weapon 3 supply is low, it might be advisable to choose weapon 1, even when weapon 2s were available. These kinds of subleties are currently being investigated.

ATTACK-FORCE ATTRITION

A factor not taken into account in either the SIAM or SEAM models is the attrition rate of the attacking forces. In particular, it is often argued that the attrition rate will be lower when special-purpose weapons are used. The problem studied can be enlarged to include attrition as a factor in either model. The difficulty, however, is that the attrition rates are highly uncertain for almost every situation. For this reason, it is unlikely that the basic concepts that emerged in this work will change considerably when attrition is included. Methods for including attrition are presently being investigated.

SUPPLY NETWORKS

As discussed in the first section, the methodology introduced here can be used at several different levels. For the problems associated with supplying a region or an individual aircraft carrier, one important complication is the existence of the supply network. For example, consider the case of an aircraft carrier about to be loaded for a deployment. Although the primary source of weapons is the carrier's own magazine, normally a supply ship travels with the carrier. Further away (in both time and space) are land-based intermediate supply depots (ISDs). Finally, there are supply bases in the continental U.S. (CONUS). Supply ships can be used to load weapons at the ISD and deliver them to a task force. The ISD can then be resupplied from CONUS. A delay is associated with each of the resupply actions. The existence of a supply network mitigates some of the negative effects of mixes composed mainly of special-purpose weapons, as long as the pace of operations is slow enough that the carrier does not run out of weapons. To model the effects of the supply network, a dynamical version of the problem considered in this analysis must also be considered. A discrete time formulation is a natural one in which the total value is the sum of values in each "period." The mix of components on the carrier will then increase due to supply actions and decrease due to attacks.

One of the most interesting questions associated with the use of a supply network involves the initial allocation of components. That is, how should an initial allocation be spread over CONUS and various ISDs (located in widely separated geographic areas) so as to optimize the ultimate value of a mix? Such questions are currently under investigation.

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APPENDIX

A MODEL WITH KNOWN TARGETS

APPENDIX

A MODEL WITH KNOWN TARGETS

This appendix describes a model aimed at the problem of ordnance acquisition and allocation when the targets are known. Such a model chooses only special-purpose (SP) weapons. The proof presented here presumes that both the target vector {T} and the kill probabilities P_{ij} are known with certainty. If the kill probabilities are uncertain, the methodology in the main text must be used, even if {T} were known. This discussion proves that if an SP weapon exists for each target type, the solution to WAP1 of the main text consists of only SP weapons; no general-purpose weapons will ever be selected. The analysis is based on the following generalization of WAP1 of the main text:

$$\max_{\{n_{ij}\}} \sum_{j=1}^{Q} f_j\left[T_j,\ 1 - \exp\left(-\sum_{i=1}^{W} n_{ij}P_{ij} \,/\, T_j\right)\right] \tag{A-1}$$

such that

$$\sum_{j=1}^{Q} \sum_{i=1}^{W} n_{ij} = N \ .$$

Here $f_j(\cdot\,,\,\cdot)$ is an arbitrary function of its arguments. If $f_j(x,y) = xy$, then equation A-1 corresponds to the expected number of targets destroyed. If $f_j(x,y) = 1 - [1 - y\,(1 - \exp(-\rho_j))]^x$, then equation A-1 corresponds to an incorrectly posed version of equation 21 of the main text. Other choices of f_j would correspond to other MOEs. Equation A-1 corresponds to an optimization problem with no uncertainty in the target vector. It will be shown that the solution to equation A-1, denoted by $\{n^*_{ij}\}$, never involves GP weapons. For this reason, a mix $\{n^*_{ij}\}$ for each possible target vector cannot be determined and then averaged over target vectors; this procedure is completely wrong.

To show that the solution to equation A-1 never involves GP weapons, begin by forming the Lagrangian

$$L = \sum_{j=1}^{Q} f_j\left[T_j,\ 1 - \exp\left(-\sum_{i=1}^{W} n_{ij}P_{ij} \,/\, T_j\right)\right]$$

$$- \lambda\left(\sum_{i=1}^{W}\ \sum_{j=1}^{Q} n_{ij} - N\right) + \sum_{i=1}^{W}\ \sum_{j=1}^{Q} \mu_{ij}n_{ij} \quad . \tag{A-2}$$

Here λ and the μ_{ij} are multipliers used to ensure that the constraints are satisfied. In particular, $\mu_{ij} \geq 0$ with $\mu_{ij} > 0$ implying that $n^*_{ij} = 0$ [A-1]. Taking the derivative of L with respect to n_{ij} and setting it equal to 0 gives

$$f'_j\left[T_j,\ 1 - \exp\left(-\sum_{i=1}^{W} n_{ij}P_{ij} \,/\, T_j\right)\right]\frac{P_{ij}}{T_j} \exp\left(-\sum_{i=1}^{W} n_{ij}P_{ij} \,/\, T_j\right) - \lambda + \mu_{ij} = 0 \quad . \tag{A-3}$$

In equation A-3, $f'_j(x, y)$ denotes the derivative of $f_j(x, y)$ with respect to its second argument. Next observe that for fixed j, the quantity

$$C_j = f'_j\left[T_j,\ 1 - \exp\left(-\sum_{i=1}^{W} n_{ij}P_{ij} \,/\, T_j\right)\right]\frac{1}{T_j} \exp\left(-\sum_{i=1}^{W} n_{ij}\ P_{ij} \,/\, T_j\right) \tag{A-4}$$

is constant over i. It is positive if $f'_j(\cdot, \cdot)$ is positive--a reasonable assumption. Then equation A-3 becomes

$$\lambda = C_j\ P_{ij} + \mu_{ij} \quad . \tag{A-5}$$

Now order the effectiveness of the weapons so that

$$P_{1j} > P_{2j} > \ldots > P_{Wj} \quad . \tag{A-6}$$

According to equation A-6, weapon type 1 is most effective, weapon type 2 is next, and so on. It is certainly possible that i and m exist so that $P_{ij} = P_{mj}$. This is a special case of the result treated here and a similar kind of calculation applies. In fact, the only case of real import is if $P_{1j} = P_{2j} > P_{3j} > \ldots$, in which case weapons 1 and 2 are special-purpose weapons.

The solution to equation A-1 will always have $n^*_{1j} > 0$. Observe that if N is increased by 1, the functional is increased the most by the addition of an SP weapon. An inductive argument shows that $n^*_{1j} > 0$.

Because $n^*_{1j} > 0$, $\mu_{1j} = 0$. Thus for any other i,

$$\lambda = P_{1j}c_j = P_{ij}c_j + \mu_{ij} \quad , \tag{A-7}$$

so that

$$\mu_{ij} = c_j\,(P_{1j} - P_{ij}) > 0 \quad . \tag{A-8}$$

Because $\mu_{ij} > 0$, $n^*_{ij} = 0$ for $i = 2, 3, \ldots, W$.

Consequently, the solution to equation A-1 involves only SP weapons, and it is the only solution to the optimization problem.

REFERENCE

[A-1] Wismer, D.A., and Chattergy, R. Introduction to Nonlinear Optimization. New York: North Holland, 1978

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